Mid Semester Examination

Electrodynamics
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February 20$^{\text{th}}$, 2025 (Morning Session)
Duration: 120 minutes.
Total points: 40

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. (a) A closed volume $\mathbb{V}$, bounded by a surface $\mathbb{S}$, carries a uniform volume charge density ρ. Show that the electric field created by this charge density can be written purely as a surface integral:

$$\vec{E}(\vec{r}) = \frac{\rho}{4\pi\epsilon_0} \oint_{\mathbb{S}} \frac{d\vec{S}'}{|\vec{r} - \vec{r'}|}$$

   (b) Imagine a charge distribution which is localized (i.e., extends over a finite closed volume), but otherwise arbitrary. Show that the electric field due to such a charge distribution can be written as

$$\vec{E}(\vec{r}) = \frac{-1}{4\pi\epsilon_0} \int_{\mathbb{V}} d^3r' \frac{\nabla_{\vec{r'}} \rho(\vec{r'})}{|\vec{r} - \vec{r'}|}$$

   5 + 5 = 10 points

2. Imagine you have two concentric spherical shells of radii a and b ($a < b$) built out of conducting materials. At electrostatic equilibrium, the shells are equipotentials. Imagine both of them are at zero potentials (which can be arranged by connecting both of them to the earth by perfectly conducting wires (this is called grounding). Now a point charge q is placed at a distance r from the common center ($a < r < b$), in the vacuum space between the two shells. This will induce charges on the surface of the two shells. Use Green's reciprocity theorem to find the induced charges on both spherical shells.

   10 points

3. Consider a sphere with radius R with the following charge density on its surface:

$$\sigma(r, \theta, \phi) = \sigma_0 \sin^2 \theta$$

Find the potential $\varphi(r, \theta, \phi)$ everywhere. (Hint: Think about trying an expansion in the proper co-ordinate system).
10 points

4. Find the net force (magnitude and direction) that is exerted by the southern hemisphere of a uniformly charged solid sphere of radius R and total charge Q on its own northern hemisphere.

10 points